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ORIGINAL ARTICLE

A method for estimating the fresh water–salt water interface with hydraulic heads in a coastal aquifer and its application

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Abstract Examining the descriptions of piezometric heads at two points in both the salt water and fresh water zones reveals that when the groundwater flow system is in steady state and satisfies the Dupuit assumption, the location of the fresh water–salt water interface in a homogeneous, isotropic, and unconfined coastal aquifer can be estimated based on a piezometric head of fresh water at a point in the fresh water zone (from the water table to the interface) vertically lined up with a piezometric head of salt water at a point in the salt water zone (from the interface down). Research shows that the new method is a general relation and that both the Hubbert relation describing the location of the interface and the Ghyben–Herzberg relation are special cases of this method. The method requires two piezometric wells to be close to each other and each tapping into a different zone. Measurements of piezometric heads at a well cluster consisting of piezometric wells tapping separately into fresh water and salt water zones near Beihai, China at 5-day intervals for 15 months are used to illustrate the estimation of interface location. The depth of the interface for well H5 ranges from 32 to 72 m below the sea level.

1. Introduction

Fresh groundwater is an important source of water supply in coastal areas. Knowing the temporal and spatial evolution of the fresh water–salt water interface is significant for groundwater development and prevention of sea water intrusion and for understanding the vulnerability of a coastal environment. Many hydrogeological studies have discussed and used models describing a sharp interface between fresh water and salt water in coastal areas, especially in coastal aquifers with a narrow transition zone between fresh water and salt water, or in coastal zones where a preliminary examination of the relation between the two water bodies is needed (Glover, 1959; Fetter, 1972; Bear, 1979; Essaid, 1990; Croucher and

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O'Sullivan, 1995; Izuka and Gingerich, 1998; Cheng and Ouazar, 1999; Feseker, 2007; Tang et al., 2007; Kim et al., 2007). Previous studies in determining the interface are based mainly on the water table in the fresh water zone with the Ghyben–Herzberg relation (ven der Veer, 1977; Reilly and Goodman, 1985; Inouchi et al., 1985; Isaacs and Hunt, 1986; Ledoux et al., 1990; Moore et al., 1992; Huyakorn et al., 1996; Person et al., 1998; Maas, 2007). By defining the piezometric heads of two fluids of different densities at a point in either fluid region, Hubbert (1940) presented a relation describing the location of the interface that relies on the piezometric heads of the two fluids to be at the same point on the interface. Izuka and Gingerich (1998) presented a method to estimate the interface depth based on vertical head gradients using water-level measurements during drilling of a partially penetrating well above the interface. Kim et al. (2007) estimated the interface depth using two sets of pressure data obtained from both fresh water and salt water zones in a single borehole. Zhou et al. (2008) and Zhou (2008) introduced methods for determining the location of the fresh water–salt water interface in coastal zones using hydraulic heads. In this paper, application of the developed method is given to illustrate the estimation of the interface, with hydraulic heads measured in the fresh water zone and the salt water zone in the coastal aquifer near Beihai, China, where the groundwater flow system is thought to be in a steady state and meets the Dupuit assumption.

2. Method

As shown in Fig. 1, it is assumed that (1) the coastal unconfined aquifer is homogeneous and isotropic; (2) groundwater flow discharges into the sea and is in steady state and; (3) the groundwater flow system satisfies the Dupuit assumption. A vertical line AE intersects the water table at point D, any point in the fresh water zone at C, the interface at point B, and any point in the salt water zone below the interface at point A. As illustrated in Fig. 1, h_s and h'_s stand for the elevations of the piezometric heads of salt water measured at point A above the mean sea level and above an arbitrary datum respectively; h_f and h'_f are the elevations of the water table at point D above the sea level and an arbitrary datum respectively; M and M' are the depth of the interface below sea level and the elevation of the interface above the datum respectively; M_s and M'_s are the depth of point A below the sea level and the elevation of point A above the datum respectively; M_f and M'_f are the depth of point C below the sea level and the elevation of point C above the datum respectively; ρ_s and ρ_f are densities of salt water and fresh water respectively. Fig. 1 shows that $h'_s = h_s + H_0$, $h'_f = h_f + H_0$, and $M' = H_0 - M$, where H_0 is the elevation of the sea level above the datum.

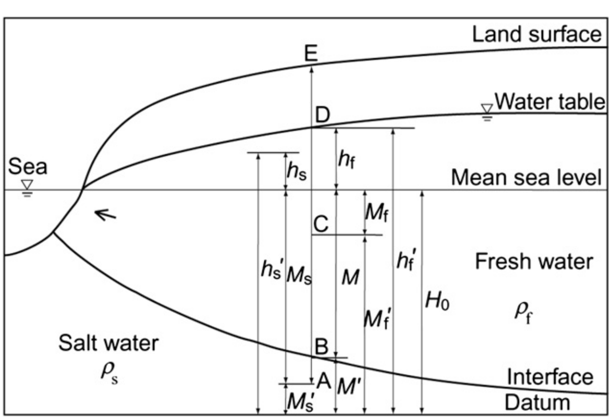

Figure 1 Schematic diagram showing a fresh water–salt water interface that generally occurs in a homogeneous, isotropic, and unconfined aquifer in a coastal zone.

Let us examine the piezometric heads at two points separately in the fresh water and salt water zones in the same vertical line AE in the coastal aquifer shown in Fig. 1. The piezometric head of fresh water at any given point C in the fresh water zone above the arbitrary datum (φ'_{Cf}) can be described as

$$\varphi'_{Cf} = \frac{P_C}{\rho_f g} + M'_f = \frac{\left(h'_f - M'_f\right)\rho_f g}{\rho_f g} + M'_f = h'_f, \tag{1}$$

where P_C is the pressure at point C and g is the gravitational acceleration.

For point A in the salt water zone on the same vertical line, we can also describe the piezometric head of salt water above the datum (φ'_{As}) as

$$\varphi'_{As} = \frac{P_A}{\rho_s g} + M'_s = \frac{(M' - M'_s)\rho_s g + \left(M'_f - M'\right)\rho_f g + P_C}{\rho_s g} + M'_s, \tag{2}$$

where P_A is the pressure at point A. P_C in Eq. (2) can be expressed as

$$P_C = \left(\varphi'_C - M'_f\right)\rho_f g. \tag{3}$$

Substituting Eq. (3) into Eq. (2) gives

$$\varphi'_{As} = (M' - M'_s) + \left(M'_f - M'\right)\frac{\rho_f}{\rho_s} + \left(\varphi'_{Cf} - M'_f\right)\frac{\rho_f}{\rho_s} + M'_s. \tag{4}$$

Rearrangement of Eq. (4) yields

$$\varphi'_{As} = \frac{\rho_s - \rho_f}{\rho_s}M' + \frac{\rho_f}{\rho_s}\varphi'_{Cf} \tag{5}$$

From Eq. (5), we obtain:

$$M' = \frac{\rho_s}{\rho_s - \rho_f}\varphi'_{As} - \frac{\rho_f}{\rho_s - \rho_f}\varphi'_{Cf} = (1+\delta)\varphi'_{As} - \delta\varphi'_{Cf}, \tag{6}$$

where

$$\delta = \frac{\rho_f}{\rho_s - \rho_f}. \tag{7}$$

Note that $\varphi'_{Cf} = \varphi_{Cf} + H_0$, $\varphi'_{As} = \varphi_{As} + H_0$, and $M' = H_0 - M$, where φ_{Cf} and φ_{As} are the piezometric head of fresh water measured at point C and the piezometric head of salt water measured at point A above the mean sea level, we can also obtain the following relation from Eq. (6) when the sea level is used as the datum:

$$M = \frac{\rho_f}{\rho_s - \rho_f}\varphi_{Cf} - \frac{\rho_s}{\rho_s - \rho_f}\varphi_{As} = \delta\varphi_{Cf} - (1+\delta)\varphi_{As}. \tag{8}$$

Eq. (8) or (6) gives the depth of the interface below sea level or the elevation of the interface above the datum, which is determined by the piezometric head of salt water at point A in the salt water zone, φ_{As} or φ'_{As} , and the piezometric head of fresh water at point C on the same vertical line in the fresh water zone, φ_{Cf} or φ'_{Cf}. These values are relatively easy to measure. This method requires two piezometric wells near each other tapping separately into the salt water and fresh water zones.

If points A and C tap simultaneously at the same point B on the interface, Eq. (6) or (8) becomes the Hubbert relation describing

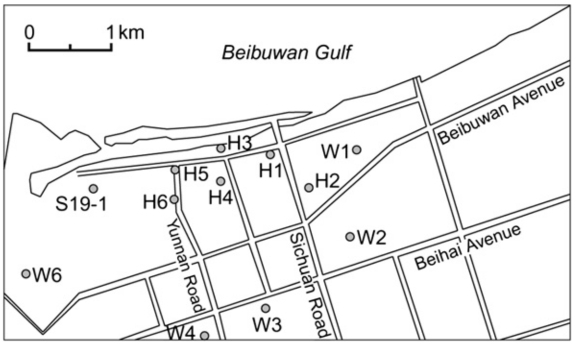

Figure 2 Map showing location of the observation well clusters in the northwestern coastal area of Beihai, Guangxi, China.

the relationship among the fresh water head, salt water head, and the location of a point on the fresh water–salt water interface. The Hubbert relation is difficult to apply since both the fresh water head and the salt water head at the same point on the interface are not known if the interface's location is previously unknown.

Since $\varphi'_{As} = h'_s$ at point A and $\varphi'_{Cf} = h'_f$ in Eq. (1), from Eq. (6) we obtain:

$$M' = \frac{\rho_s}{\rho_s - \rho_f} h'_s - \frac{\rho_f}{\rho_s - \rho_f} h'_f = (1+\delta)h'_s - \delta h'_f. \tag{9}$$

From Eq. (9) or (8) we can also obtain the following relation when the sea level is used as the datum:

$$M = \frac{\rho_f}{\rho_s - \rho_f} h_f - \frac{\rho_s}{\rho_s - \rho_f} h_s = \delta h_f - (1+\delta)h_s. \tag{10}$$

Eq. (10) or (9) gives the depth of the interface below sea level or the elevation of the interface above the datum, which is determined by piezometric head of salt water at point A in the salt water zone, h_s or h'_s , and the water table of fresh water on the same vertical line in the fresh water zone, h_f or h'_f. If point C is on the water table and the piezometric head of salt water at point A is identical to the sea level, *i.e.,* $h_s = 0$ or $h'_s = H_0$, Eq. (10) or (9) reduces into the Ghyben–Herzberg relation:

$$M = \delta h_f \tag{11}$$

or

$$M' = (1+\delta)H_0 - \delta h'_f. \tag{12}$$

Since points A and C are arbitrary, Eq. (8) or (6) is a general relation that can be used to estimate the location of the fresh water–salt water interface when the previously mentioned assumptions are met in coastal zones. The Hubbert relation is a special case of Eq. (8) or (6) and the Ghyben–Herzberg relation is a special case of Eq. (10) or (9).

3. Application

The groundwater system in the Beihai coastal region of southern Guangxi, China, is used to illustrate the application of the developed principle. The coastal plain is underlain by Quaternary and Neogene unconsolidated sediments consisting of sand with gravel and scattered lenses of clay or sandy clay; the total thickness ranges from 5 to 350 m. The unconsolidated sediments in the southern part of the region can roughly be grouped into one unconfined aquifer and three confined aquifers. Because of relief on the basement–sediment interface, only one unconfined aquifer and one confined aquifer are present in the northern part of the study area. Hydraulic connections exist among the aquifers, especially among the confined aquifers due to the semi-perviousness and termination of clay. The three confined aquifers, therefore, can be treated as one confined aquifer. A detailed description of the hydrogeological setting was given by Zhou et al. (2000, 2006).

Eleven clusters of wells were drilled in the northwestern coastal zone in the city of Beihai (Fig. 2, excerpt well S19-1). Each of the well clusters has three piezometric wells of different depths in very close proximity. The three piezometric wells at well cluster H5 from upper to lower zones were named H5-1, H5-2,

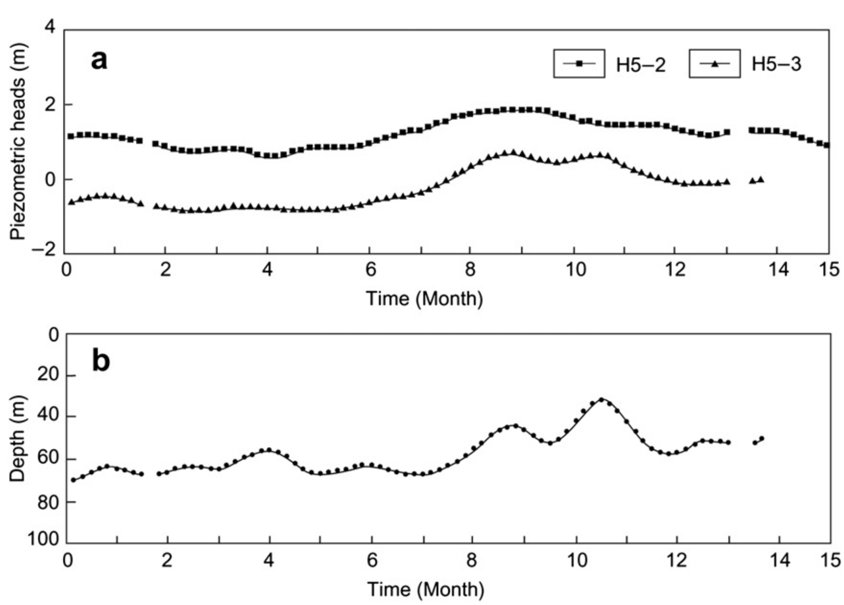

Figure 3 Changes in (a) average piezometric heads in wells H5-2 and H5-3 and (b) calculated depths of the fresh water–salt water interface below sea level from January, 1996 to March, 1997.

and H5-3. Piezometric wells H5-2 and H5-3 are thought to tap separately into the fresh water and salt water zones. Measurements of the piezometric heads were carried out once every five days from January, 1996 to March, 1997. During the measurement period, the piezometric heads at well H5-3 tapping into the salt water zone were lower than the mean sea level in the dry season and higher than the sea level in the rainy season. This observation may suggest that the change in location of the fresh water–salt water interface depends on both the piezometric head of fresh water above the interface and the piezometric head of salt water below the interface. Both heads should be considered in estimating the location of the interface.

For $\rho_f = 1.0$ and $\rho_s = 1.025$, Eq. (8) becomes:

$$M = 40\varphi_{Cf} - 41\varphi_{As}, \tag{13}$$

assuming that the aquifer near well cluster H5 satisfies the previously mentioned assumptions.

Eq. (13) is used to estimate the interface depth in the northwestern coastal zone in Beihai, with the assumption that a sharp interface exists between the fresh water and salt water zones. Piezometric heads at well cluster H5 are used since the aquitard between the unconfined and confined aquifers terminates near this well cluster. Piezometric head relative to sea level at H5-2 represents the hydraulic head of fresh water above the interface (φ_{Cf}) and piezometric head relative to sea level at H5-3 represents the hydraulic head of salt water at a point in the salt water zone below the interface (φ_{As}). Data from piezometric heads at H5-2 and H5-3 are first smoothed out using the Haning formula of running average to eliminate minor fluctuations in the measured piezometric heads caused by tide (Fig. 3a). The estimated interface depths at well H5 according to Eq. (13) at every time step during the measurement period are shown in Fig. 3b, and the depths range from 32 to 72 m below the sea level. The interface was lower in dry season than in rainy season due to the raising of piezometric heads in salt water. The results suggest that both the fresh water head in the fresh water zone and salt water head in the salt water zone have an important bearing on the interface location.

4. Conclusion

When the groundwater flow in a coastal aquifer is in a steady state and satisfies the Dupuit assumption, the location of the fresh water–salt water interface in a homogeneous, isotropic, and unconfined coastal aquifer can be estimated using a piezometric head of fresh water measured at any point in the fresh water zone (from the interface to the water table) in combination with a piezometric head of salt water measured at any point in the salt water zone (from the interface down) on the same vertical line. The Hubbert relation describing the interface location is a special case with two points simultaneously tapping the same point on the interface. The Ghyben–Herzberg relation is also a special case with the fresh water piezometric head tapping the water table and the salt water piezometric head being at the sea level.

Measurements at 5-day intervals from piezometric heads of salt water and piezometric heads of fresh water at well cluster H5 near Beihai, China, show that the salt water heads can be lower or higher than the mean sea level. At every measured time, the measurements of piezometric heads are used to estimate the location of the fresh water–salt water interface. During the measurement period, the calculated interface depth for well H5 range from 32 to 72 m below the sea level. Error may exist if the Dupuit assumption is not satisfied, especially approaching or far from the coast. This method requires the two nearest piezometric wells to penetrate separately into the fresh water and salt water zones.

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